Learning Strikes Again: the Case of the DRS Signature Scheme

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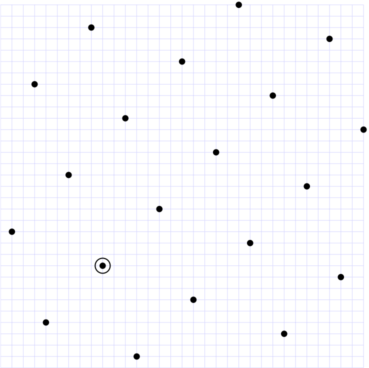
- Target: DRS — a NIST lattice-based signature proposal
- Techniques: learning & lattice
  - Statistical learning $\Rightarrow$ secret key information leaks
  - Lattice techniques $\Rightarrow$ better use of leaks
- They claim that Parameter Set-I offers **at least 128-bits** of security. We show that it actually offers **at most 80-bits** of security!

Outline

1. Background
2. DRS signature
3. Learning secret key coefficients
4. Exploiting the leaks
5. Countermeasures

Outline

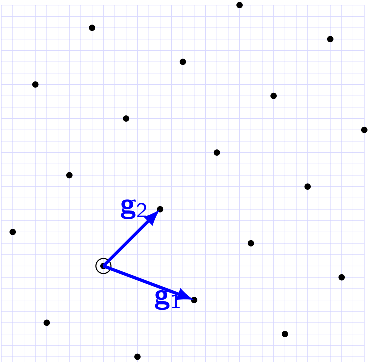
Lattice



Definition

A lattice $\mathcal{L}$ is a discrete subgroup of $\mathbb{R}^m$.

Lattice



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A lattice is generated by its basis $\mathbf{G} = (\mathbf{g}_1, \cdots, \mathbf{g}_n) \in \mathbb{R}^{n \times m}$, e.g. $\mathcal{L} = \{\mathbf{x}\mathbf{G} \mid \mathbf{x} \in \mathbb{Z}^n\}$.

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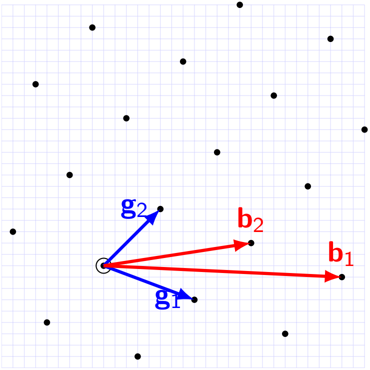

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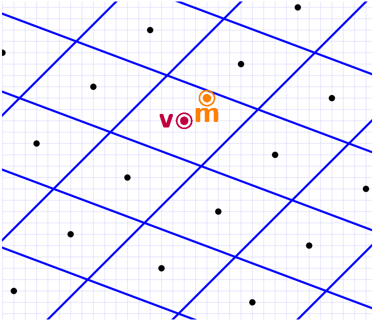
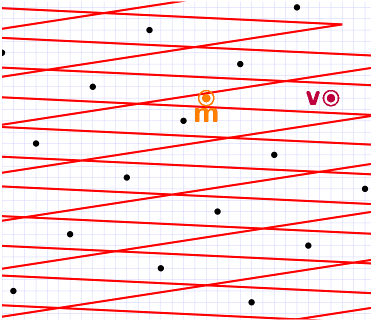
$\mathcal{L}$ has infinitely many bases

$\mathbf{G}$ is good, $\mathbf{B}$ is bad.

Finding close vectors

Each basis defines a parallelepiped $\mathcal{P}$.





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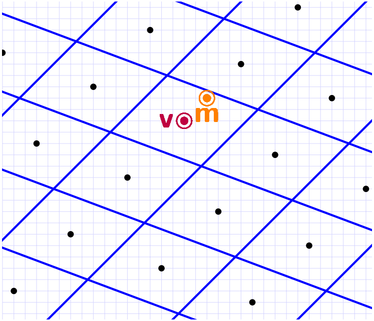
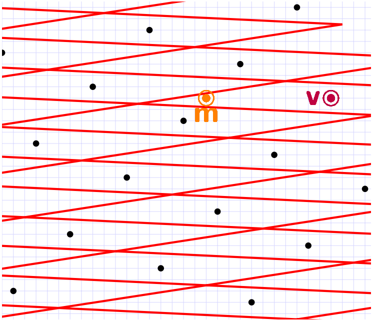




Babai's round-off algorithm outputs $\mathbf{v} \in \mathcal{L}$ such that $\mathbf{v} - \mathbf{m} \in \mathcal{P}$.

GGH & NTRUSign schemes

Public key: $\mathbf{P}$, secret key: $\mathbf{S}$

Sign

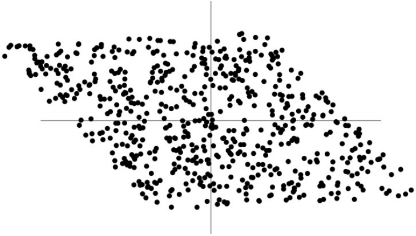
1. Hash the message to a random vector $\mathbf{m}$
2. Round $\mathbf{m}$ (using $\mathbf{S}$) to $\mathbf{v} \in \mathcal{L}$

Verify

1. Check $\mathbf{v} \in \mathcal{L}$ (using $\mathbf{P}$)
2. Check $\mathbf{v}$ is close to $\mathbf{m}$

GGH & NTRUSign are insecure!

$\mathbf{v} - \mathbf{m} \in \mathcal{P}(\mathbf{S}) \Rightarrow (\mathbf{v}, \mathbf{m})$ leaks some information of $\mathbf{S}$.



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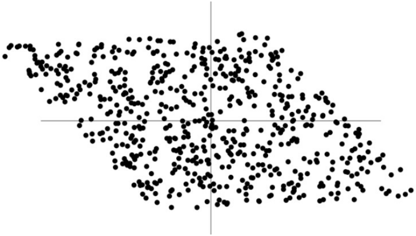

GGH and NTRUSign were broken by “learning the parallelepiped” [NR06].

Some countermeasures were also broken by a similar attack [DN12].

Countermeasures

Let us focus on Hash-then-Sign approach!

Provably secure method [GPV08]

- rounding based on Gaussian sampling
- $\mathbf{v} - \mathbf{m}$ is independent of $\mathbf{S}$

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Heuristic method [PSW08]

- rounding based on CVP w.r.t ℓ_∞-norm
- the support of $\mathbf{v} - \mathbf{m}$ is independent of $\mathbf{S}$
- DRS [PSDS17] is an instantiation, submitted to the NIST.

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DRS

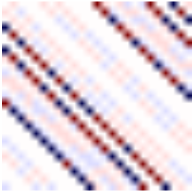
DRS $=$ **D**iagonal-dominant **R**eduction **S**ignature

DRS

DRS = **D**iagonal-dominant **R**eduction **S**ignature

Parameters: (n, D, b, N_b, N_1)

- n : the dimension
- D : the diagonal coefficient
- b : the magnitude of the large coefficients (*i.e.* $\{\pm b\}$)
- N_b : the number of large coefficients per row vector
- N_1 : the number of small coefficients (*i.e.* $\{\pm 1\}$) per row vector

$$\mathbf{S} = \begin{pmatrix} D & & & \\ & D & & \\ & & \ddots & \\ & & & D \end{pmatrix} +$$ 

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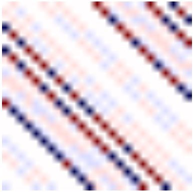

$\leftarrow \begin{cases} D > b \cdot N_b + N_1; \\ \text{absolute circulant} \end{cases}$

Message reduction algorithm

Input: a message $\mathbf{m} \in \mathbb{Z}^n$, the secret matrix $\mathbf{S}$
Output: a reduced message $\mathbf{w}$ such that $\mathbf{w} - \mathbf{m} \in \mathcal{L}$ and $\|\mathbf{w}\|_\infty < D$

$\mathbf{w} \leftarrow \mathbf{m}$, $i \leftarrow 0$
repeat
    $\mathbf{w} \leftarrow \mathbf{w} - \lfloor \frac{w_i}{D} \rfloor_{\to 0} \cdot \mathbf{s}_i$
    $i \leftarrow (i+1) \bmod n$
until $\|\mathbf{w}\|_\infty < D$
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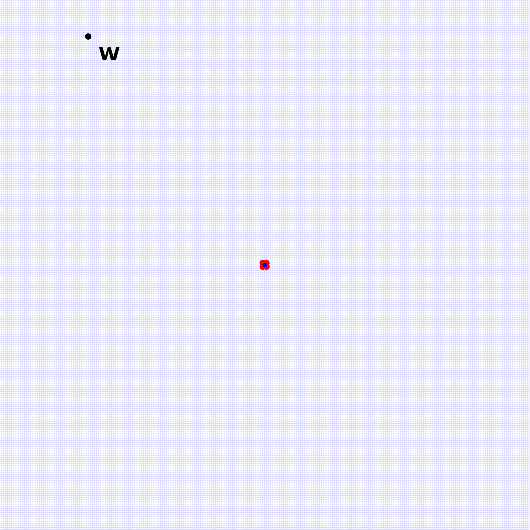

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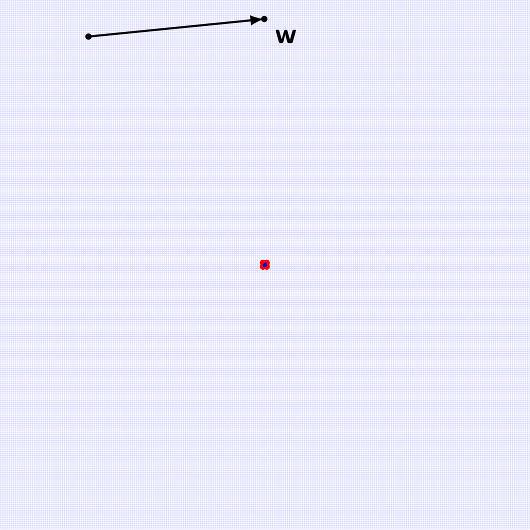

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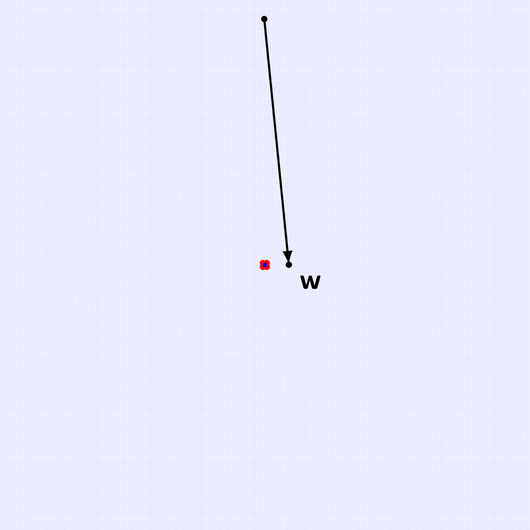

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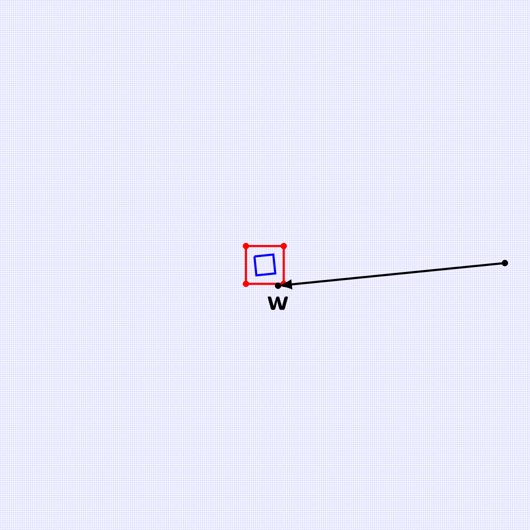

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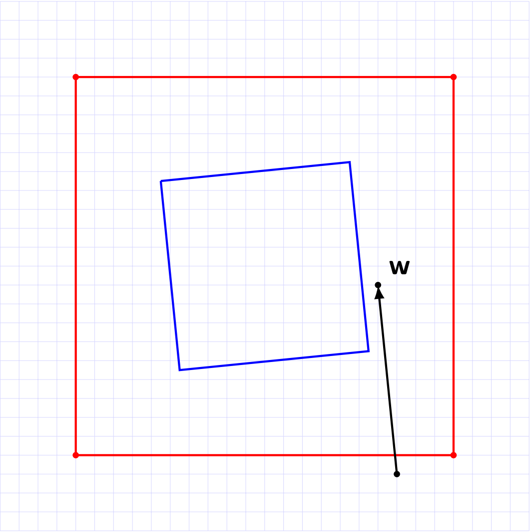

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- w_i decreases a lot
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&< \|\mathbf{w}\|_1 \qquad \text{(diagonal dominance)}
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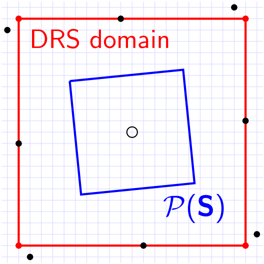
$\Rightarrow$ message reduction always terminates!

Resistance to NR attack

The support of $\mathbf{w}$: $(-D, D)^n$



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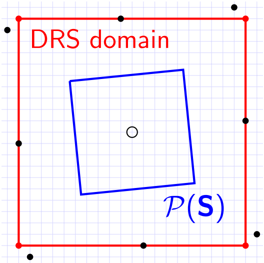

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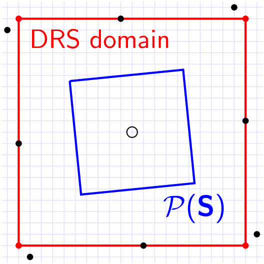

The support is "zero-knowledge", but **maybe the distribution is not!**

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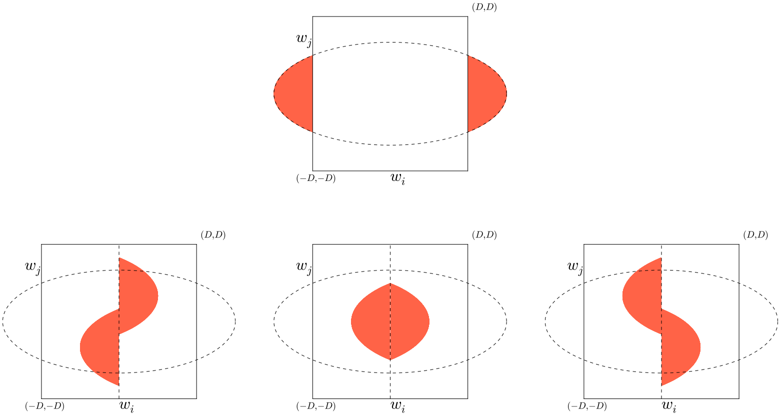
Intuition



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$\Rightarrow$ $\mathbf{S}_{i,j}$ should be **strongly related** to $W_{i,j}$ (the distribution of (w_i, w_j)) !

Figure out the model

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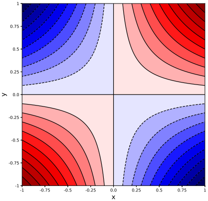
$\Rightarrow$ **Search for the best linear fit f ?**

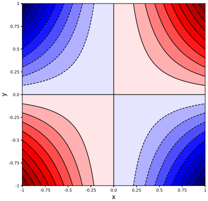
Search space for all linear f: too large!
$\Rightarrow$ **choose some features $\{f_i\}$ and search in $\mathrm{span}(\{f_i\})$, i.e. $f = \sum x_\ell f_\ell$**

Lower degree moments:

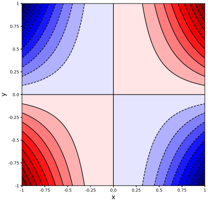
$f_1(W) = \mathbb{E}(w_i w_j)$



$f_2(W) = \mathbb{E}(w_i \cdot |w_i|^{1/2} \cdot w_j)$



$f_3(W) = \mathbb{E}(w_i \cdot |w_i| \cdot w_j)$



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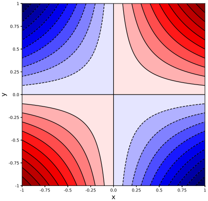
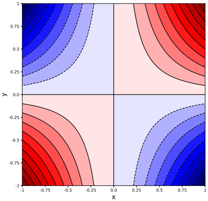
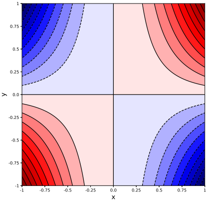
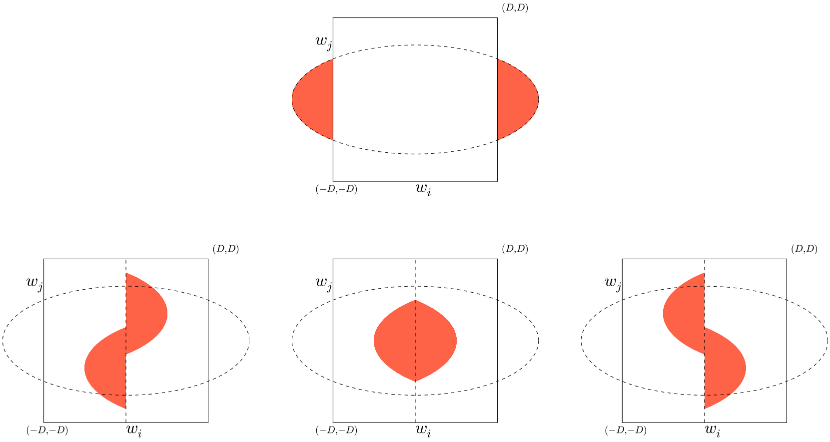






Not enough!

Training — feature selection



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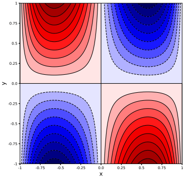
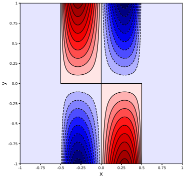
Training — feature selection

Pay more attention to the central region (i.e. $|w_i|$ small).

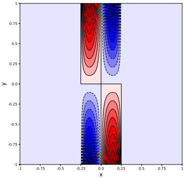
$$f_4 = \mathbb{E}(w_i(w_i-1)(w_i+1)w_j)$$



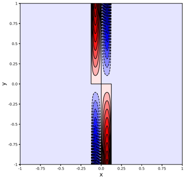
$$f_5 = \mathbb{E}(2w_i(2w_i-1)(2w_i+1)w_j \mid |2w_i| \leq 1)$$



$$f_6 = \mathbb{E}(4w_i(4w_i-1)(4w_i+1)w_j \mid |4w_i| \leq 1)$$



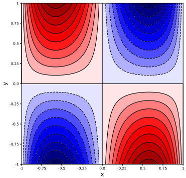
$$f_7 = \mathbb{E}(8w_i(8w_i-1)(8w_i+1)w_j \mid |8w_i| \leq 1)$$



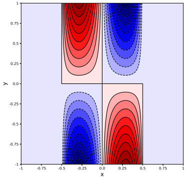
Training — feature selection

Pay more attention to the central region (i.e. $|w_i|$ small).

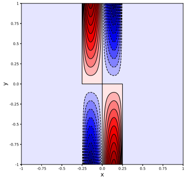
$$f_4 = \mathbb{E}(w_i(w_i-1)(w_i+1)w_j)$$



$$f_5 = \mathbb{E}(2w_i(2w_i-1)(2w_i+1)w_j \mid |2w_i| \le 1)$$



$$f_6 = \mathbb{E}(4w_i(4w_i-1)(4w_i+1)w_j \mid |4w_i| \le 1)$$



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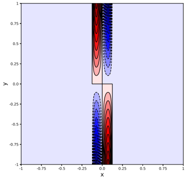

Together with transposes (i.e. $f^t(w_i, w_j) = f(w_j, w_i)$), we finally selected $7 \times 2 - 1 = 13$ features in experiments.

Training — model construction

$\mathbf{S}_{i,j}$ seems easier to learn when $(i - j \bmod n)$ is smaller.

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- 30 instances and 400 000 samples per instances
- 38 core-hours

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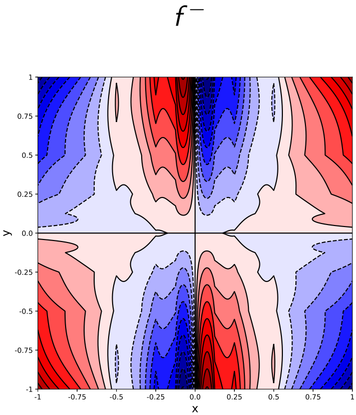
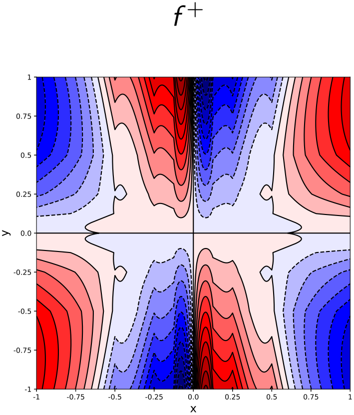
The models



f^-
1
0.75
0.5
0.25
0.0
-0.25
-0.5
-0.75
-1
y
-1
-0.75
-0.5
-0.25
0.0
0.25
0.5
0.75
1
x

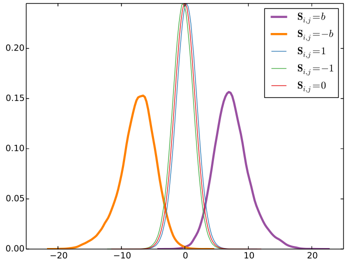


f^+
1
0.75
0.5
0.25
0.0
-0.25
-0.5
-0.75
-1
y
-1
-0.75
-0.5
-0.25
0.0
0.25
0.5
0.75
1
x

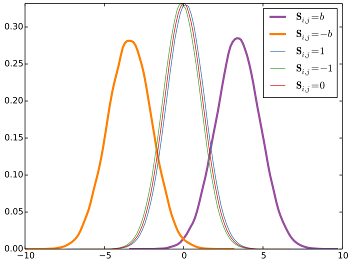
Learning

Let's learn a new $\mathbf{S}$ as $\mathbf{S}' = f(W)$!

f^-



f^+



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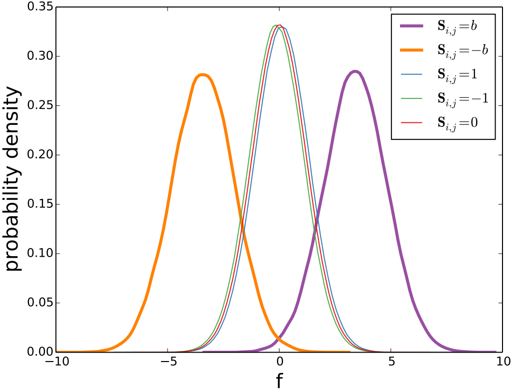

Learning

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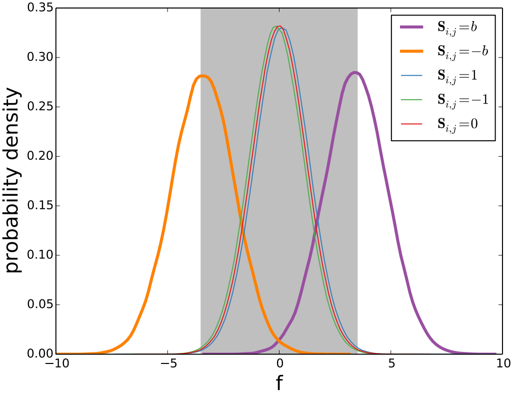
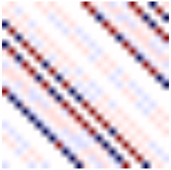
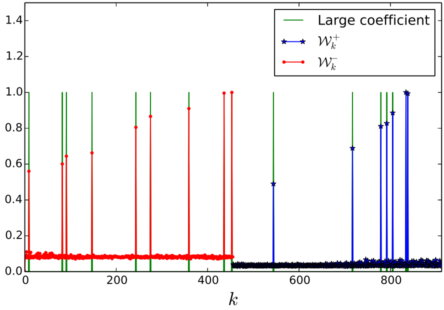


$\mathbf{S} = D \cdot I +$  is "absolute circulant"

$\Rightarrow$ more confidence via diagonal amplification

Learning — location

The weight of k-th diagonal $\mathcal{W}_k = \sum \mathbf{S}'^2_{i,i+k}$



Learning — location

#signatures	13/16	14/16	15/16	16/16
50 000	5	3	6	6
100 000	-	-	-	20
200 000	-	-	-	20
400 000	-	-	-	20

Table: Location accuracy. The column, labeled by $K/16$, shows the number of tested instances in which the largest N_b scaled weights corresponded to exactly K large coefficient diagonals.

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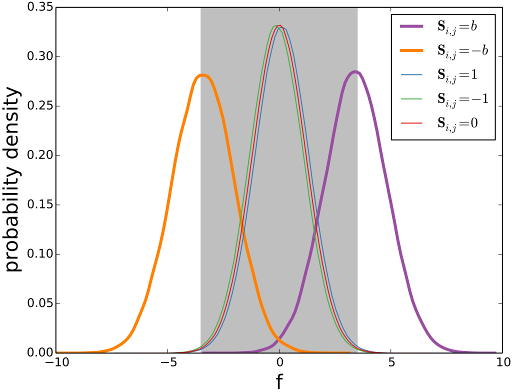
but we are still missing the signs!

Learning — sign

$\mathbf{S}_{i,j} \in \{\pm b, \pm 1, 0\}$



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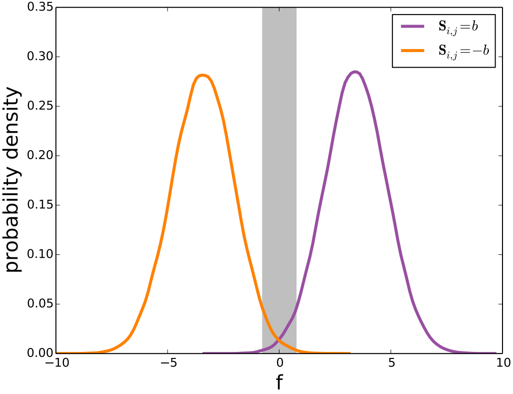

Learning — sign

#signatures	p_l	p_u	p	p_{row}
400 000	0.9975	0.9939	0.9956	0.9323
200 000	0.9920	0.9731	0.9826	0.7546
100 000	0.9722	0.9330	0.9536	0.4675
50 000	0.9273	0.8589	0.8921	0.1608

Table: Experimental measures for p_l, p_u, p and p_{row}.

$p =$ accuracy of guessing the sign of a large coefficient

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We can determine all large coefficients in one row!
However, it is still hard to learn small coefficients...

Outline

BDD & uSVP

BDD (Bounded Distance Decoding)

Given a lattice $\mathcal{L}$ and a target $\mathbf{t}$ "very close" to $\mathcal{L}$, to find $\mathbf{v} \in \mathcal{L}$ minimizing $\|\mathbf{v} - \mathbf{t}\|$.

uSVP (Unique SVP)

Given a lattice $\mathcal{L}$ with $\lambda_1(\mathcal{L}) \ll \lambda_2(\mathcal{L})$, to find its shortest non-zero vector.

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- $\lambda_1(\mathcal{L}') = \sqrt{1 + \mathrm{dist}(\mathbf{t}, \mathcal{L})^2}$
- $\mathrm{vol}(\mathcal{L}') = \mathrm{vol}(\mathcal{L})$

Solving uSVP by BKZ

Required blocksize β

[ADPS16, AGVW17]: $\sqrt{\beta/d} \cdot \lambda_1(\mathcal{L}') \leq \delta_\beta^{2\beta-d} \cdot \mathrm{vol}(\mathcal{L}')^{\frac{1}{d}}$

where $d = \dim(\mathcal{L}')$, $\delta_\beta \approx \left(\frac{(\pi\beta)^{\frac{1}{\beta}}\beta}{2\pi e}\right)^{\frac{1}{2(\beta-1)}}$ $(\beta > 50)$.

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Leaks help a lot!

Attack without leaks

- $d = n + 1$, $\lambda_1(\mathcal{L}') = \sqrt{b^2 \cdot N_b + N_1 + 1}$
- cost: $> 2^{128}$

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Improved attack with leaks

- $\mathbf{d = n - N_b},\ \lambda_1(\mathcal{L}') = \sqrt{N_1 + 1}$
- cost: 2^{73}

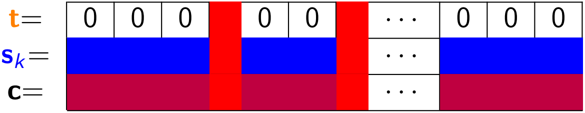
Improved BDD-uSVP attack

Red: $D, \pm b$ **(known)**, **Blue:** $0, \pm 1$ **(unknown)**

$\mathbf{t}=$	0	0	0		0	0		$\cdots$	0	0	0
$\mathbf{s}_k=$								$\cdots$			

Improved BDD-uSVP attack

Let $\mathbf{H} = \begin{pmatrix} * & & & & \\ * & * & & & \\ * & * & 1 & & \\ \vdots & \vdots & & \ddots & \\ * & * & & & 1 \end{pmatrix}$ be $HNF(\mathcal{L})$, and $\mathbf{s} = \mathbf{c}\mathbf{H}$



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$\mathbf{t}=$	0	0	0		0	0		$\cdots$	0	0	0
$\mathbf{s}_k=$								$\cdots$			
$\mathbf{c}=$								$\cdots$			

Let $\mathbf{M}$ such that

$\mathbf{tM}=$	0	0	$\cdots$	0		$= (\mathbf{0}, \mathbf{r})$
$\mathbf{s}_k\mathbf{M}=$						$= (\mathbf{b}, \mathbf{r})$
$\mathbf{cM}=$						$= (\mathbf{p}, \mathbf{r})$

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Let $\mathbf{M}^t\mathbf{H}\mathbf{M} = \begin{pmatrix} \mathbf{H}' & \\ \mathbf{H}'' & \mathbf{I} \end{pmatrix}$ and

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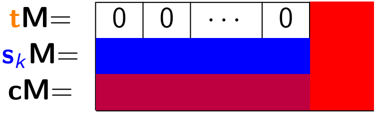
- $\dim(\mathcal{L}') = n - N_b$
- $\mathrm{vol}(\mathcal{L}') = \mathrm{vol}(\mathcal{L})$
- $\lambda_1(\mathcal{L}') = \|(\mathbf{b}, 1)\| = \sqrt{N_1 + 1}$

Once one $\mathbf{s}_i$ is recovered exactly $\Rightarrow$ all 0's in $\mathbf{S}$ are determined



$\dim = n - N_b$

Improved BDD-uSVP attack

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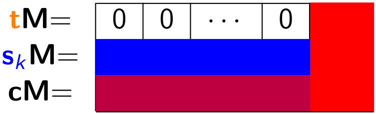

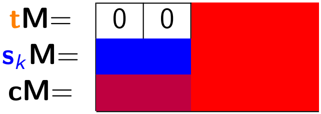

$\dim = n - N_b$

$\dim = N_1 + N_b + 1 \approx n/2$

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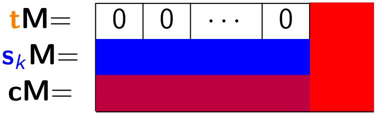

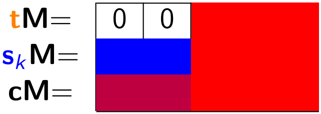

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Recovering secret matrix $\approx$ recovering a first secret.

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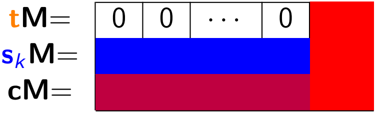

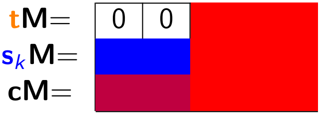

dim $= n - N_b$ dim $= N_1 + N_b + 1 \approx n/2$

Recovering secret matrix $\approx$ recovering a first secret.

Can we do better with the help of many $\mathbf{t}_k$ close to $\mathbf{s}_k$? [KF17]

Conclusion

We present a statistical attack against DRS:

- given 100 000 signatures, security is below 80-bits;
- even less with the current progress of lattice algorithms.

Outline

Modified DRS

In DRS: $\mathbf{S} = D \cdot \mathbf{I} + \mathbf{E}$ is diagonal-dominant

Version 1 [PSDS17]

- absolute circulant, $\mathbf{E}_{i,i} = 0$
- three types of coefficients $(\{0\}, \{\pm 1\}, \{\pm b\})$ with fixed numbers

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Version 2 [PSDS18]

- $\mathbf{e}_1, \cdots, \mathbf{e}_n \xleftarrow{\$} \{\mathbf{v} \mid \|\mathbf{v}\|_1 < D\}$
- variable diagonal elements

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Version 2 [PSDS18]

- $\mathbf{e}_1, \cdots, \mathbf{e}_n \xleftarrow{\$} \{\mathbf{v} \mid \|\mathbf{v}\|_1 < D\}$
- variable diagonal elements

Impact

- no circulant structure $\Rightarrow$ diagonal amplification doesn't work

Modified DRS

In DRS: $\mathbf{S} = D \cdot \mathbf{I} + \mathbf{E}$ is diagonal-dominant

Version 1 [PSDS17]

- absolute circulant, $\mathbf{E}_{i,i} = 0$
- three types of coefficients $(\{0\}, \{\pm 1\}, \{\pm b\})$ with fixed numbers

Version 2 [PSDS18]

- $\mathbf{e}_1, \cdots, \mathbf{e}_n \xleftarrow{\$} \{\mathbf{v} \mid \|\mathbf{v}\|_1 < D\}$
- variable diagonal elements

Impact

- no circulant structure $\Rightarrow$ diagonal amplification doesn't work
- coefficients are less sparsely distributed $\Rightarrow$ less confidence of guessing

Learning attack on modified DRS

We regard $\mathbf{S}_{i,j}$ as a random variable following the same distribution.
Let $\mathbf{S}'$ be the guess of $\mathbf{S}$ and N be the sample size.

Learning attack on modified DRS

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Let $\mathbf{S}'$ be the guess of $\mathbf{S}$ and N be the sample size.

As N grows, we hope

- $\mathbf{Var}(\mathbf{S}_{i,j} - \mathbf{S}'_{i,j}) < \mathbf{Var}(\mathbf{S}_{i,j}) \Rightarrow$ more confidence of guessing
- $\|\mathbf{s}_i - \mathbf{s}'_i\| < \|\mathbf{s}_i\| \Rightarrow$ guessing vector gets closer to the lattice

Some experiments on modified DRS

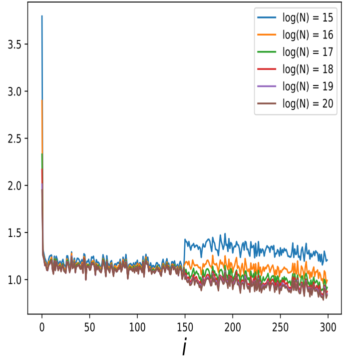
We conducted some experiments on reduced parameters.

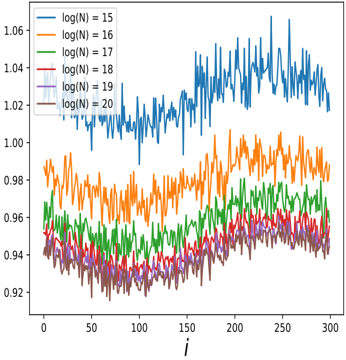
Some experiments on modified DRS

We conducted some experiments on reduced parameters.

We re-used the same approach with same features.

$$\frac{\mathbf{Var}(\mathbf{S}_{j,i+j} - \mathbf{S}'_{j,i+j})}{\mathbf{Var}(\mathbf{S}_{j,i+j})} \qquad \frac{\|\mathbf{s}_i - \mathbf{s}'_i\|}{\|\mathbf{s}_i\|}$$

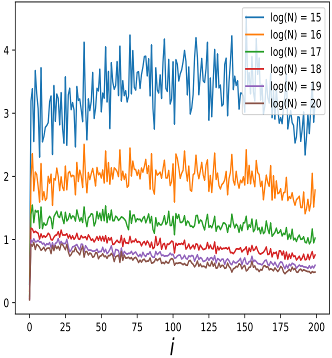




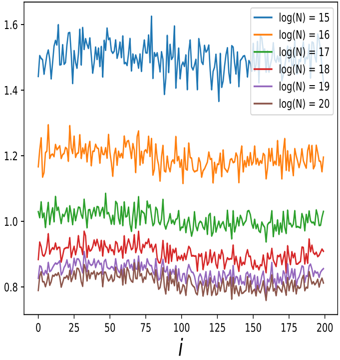
Some experiments on modified DRS

We also tried the case of n blocks and some new features.

$$\frac{\mathbf{Var}(\mathbf{S}_{j,i+j} - \mathbf{S}'_{j,i+j})}{\mathbf{Var}(\mathbf{S}_{j,i+j})}$$



$$\frac{\|\mathbf{s}_i - \mathbf{s}'_i\|}{\|\mathbf{s}_i\|}$$



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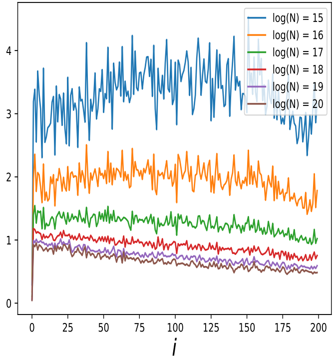
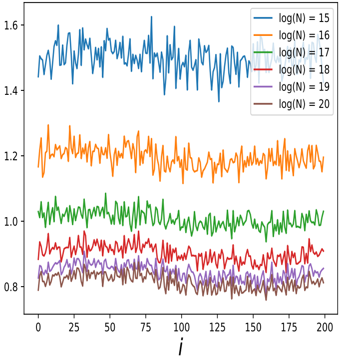
$$\frac{\mathbf{Var}(\mathbf{S}_{j,i+j} - \mathbf{S}'_{j,i+j})}{\mathbf{Var}(\mathbf{S}_{j,i+j})}$$



$$\frac{\|\mathbf{s}_i - \mathbf{s}'_i\|}{\|\mathbf{s}_i\|}$$



Further study is ongoing...

Conclusion

A leak still exists despite the new countermeasure.

Conclusion

A leak still exists despite the new countermeasure.

Work in progress

- use **timing leakage** to locate the endpoint of message reduction, then to classify samples and to choose most relevant ones

Open question

- well-designed perturbation & statistical arguments

Thank you!

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